Quantum channel for light based on integrals of motion

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A method based on integrals of motion for collective processes has been introduced to achieve physical schemes in which one of the systems is insensitive to interaction. Decoherence-free quantum channels that allow sending any state of light, particulary the Fock states, through an absorbing medium are considered as an example.

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I. INTRODUCTION

Integrals of motion may result in conserving several properties of the interacting systems, that are interesting for applications.

In the quantum informational processing the problem of integrity of the state of the physical system is significant thanks to decoherence, which destroys the state because of the irreversible interaction with environment. One of solutions of the problem is to refer to decoherence-free subspases consisting of states which have immunity to interaction and store their integrity. First decoherence-free subspaces have been introduced by Zanardi [1] by considering interaction between N two-level atoms and the multimode electromagnetic field. For this problem the subspace of atomic wave functions that are annihilated by interaction Hamiltonian has been found. For N = 2, the subspace includes only one wave function Ψ^-, which is one of the Bell states, antisymmetric with respect to the permutations of particles. As it was shown by Basharov [2] this state has immunity to decay in the collective thermostat. Some interesting examples of decoherence-free states for the spin-spin and spin-boson interactions have been considered by many authors (see, for example [3, 4]). Weinfurter et al have demonstrated experimentally a scheme of decoherence-free communication based on the four-photon polarized states of light [5].

To achieve decoherence-free spaces integrals of motion can be used. In the various processes of interactions between light and resonant and transparent media integrals of motions may result in conserving quantum correlations between modes [6, 7], that is a basis, for example, of amplification of EPR pair of continuous variables [8].

The main aim of our work is to study decoherence-free communication using integrals of motions. We show, that existence of integral of motions can establish such type of interaction between two systems that one of these systems does not change. Then we find a decoherence-free space including all states of the system in contrast to decoherence-free-subspace. For example we consider the set of schemes involving two modes of light and absorbing atoms. When there is only one of the modes it is absorbed by atoms when propagating through the medium. However despite of absorbtion it can be reproduced at the output if the second mode and some additional optical elements are introduced in the schemes. These schemes can be considered as a decoherence free cannel for sending any quantum state of light. We focus on a particular problem of sending light in the Fock state through the absorbing medium. For this case the decoherence-free channel can be achieved with the help of two non-absorbing beamsplitters and a mode in the coherence state. We consider the Fock states of light, particulary a single-photon state as they are interesting for many problems. Indeed in quantum computation many operations can be implemented by linear optics elements if the single-photon states are used. In the KLM model [9] based on linear optics logical variables 0 and 1 are encoded by the two-mode Fock states $|01\rangle, |10\rangle$. These are non-classical states of light and they have quantum correlations which have to be protected against to decoherence for successful computations.

The paper is organized as follows. First we consider a general conditions of existence of integral of motions and methods to achieve the schemes including two modes, one of which is insensitive to interaction. Then we discuss the optical schemes for sending light through absorbing medium presented by two-level atoms. As an example a problem of sending the Fock state of light is considered. The task can be accomplished when the Fock state is mixed with a mode in coherent state. Such state obtained after mixing has been demonstrated experimentally by Lvovsky [10], its properties is discussed in Appendix.

II. INTEGRAL OF MOTION

Let H_{AB} be the Hamiltonian of two interacting system A and B and $Z = Z(A, B)$ be an observable, which can depend on the variables of both systems. The evolution of operator Z is given by

$$Z' = T^{\dagger}_{AB} Z T_{AB}, \tag{1}$$

where $T_{AB} = \exp(-i\hbar^{-1}H_{AB}t)$ and we assume for simplicity that the Hamiltonian H_{AB} is time independent. If

$$[T_{AB}; Z] = 0, \tag{2}$$

then Z is an integral of motion. Condition (2) can be achieved in various cases. Let the evolution operator be

independent from the variables of one of the systems, say, B:

$$T_{AB} = T_A \otimes 1_B. \tag{3}$$

Then it is clear that any observable $Z(B)$ is integral of motion.
A nontrivial solution of Eq. (3) can be found by means of unitary transformations. Let $U(A,B)$ be a unitary operator depending on the variables of A and B. Then

$$U^\dagger(A,B)(S(A)\otimes 1_B)U(A,B) = S(A,B) \tag{4}$$

for any operator $S(A)$ and one finds Eq. (3)

$$T_{AB} = U(A,B)S(A,B)U^\dagger(A,B) = S(A)\otimes 1_B. \tag{5}$$

In accordance with (3) the evolution of the density matrix of two systems has the form

$$\rho'_{AB} = T_{AB}(\rho_A \otimes \rho_B)T^\dagger_{AB} = \rho'_A \otimes \rho_B. \tag{6}$$

Eq. (6) tells that the system B described by the density matrix ρ_B is unchanged during interaction if there is integral of motion. Note that this result is valid for any state of B. In contrast to our case Zanardi et al used another method and found a finite set of states $\{\Psi_B\}$ which are annihilated by the Hamiltinian of interaction: $H_{AB}\Psi_B = 0$.
Let the unitary operators R_A и N_B depend on the variables of A and B respectively. Then a simple transformation of Eq. (3) aries

$$(R_A \otimes N_B)^\dagger T_{AB}(R_A \otimes N_B) = T'_A \otimes 1_B, \tag{7}$$

which holds the above properties. The obtained Eq. (7) provides consideration of various physical schemes.
Eq. (5) has a simple interpretation if the operator $S(A,B)$ is unitary and hence can be associated with an Hamiltonian of interaction V_{AB}: $S(A,B) = \exp(-iV_{AB}t)$. In this case the next property aries. The existence of integral of motion allows to choose a form of interaction and to construct a scheme including three sequential transformations $U(A,B)S(A,B)U^\dagger(A,B)$, which keep the system B unchanged. Then using this scheme and unitary transformation (7) as initial resource one can construct a set of unitary equivalent schemes of the forme $U'(A,B)S'(A,B)U'^\dagger(A,B)$, where $X' = (R_A \otimes N_B)^\dagger X(R_A \otimes N_B)$ and $X = U(A,B), S(A,B)$, which accomplish the same task.
As an example consider the Bogolubov transformation of two boson operators a and b: $U^\dagger aU = ca + sb, U^\dagger bU = -sa + cb$, where $c^2 + s^2 = 1$. Then $S(ca+sb) = U^\dagger S(a)U$, where S is a unitary operator, that can describe the physical interaction between two modes that is represented by the collective operator $ca+sb$. It is clear that $T_{ab} = US(ca+sb)U^\dagger = S(a)\otimes 1_b$. Then we use (7). Let R_A be the phase shift operator: $R_a : a \to a\exp(i\mu)$, where μ is a real number and $N_B = 1$, then $T_{ab} \to T'_{ab} = U'S'(ca\exp(i\mu)+sb)U'^\dagger = S(a)\otimes 1_b$, where $U' = R^\dagger_a U R_a$. The general properties presented above can be realized for particular physical processes.

III. OPTICAL SCHEMES

We consider two optical schemes with interaction between two el. modes a and b and two-level absorbing atoms. In both schemes frequency of the mode b is resonance to the frequency of atomic transition hence this mode may be absorbed. However the mode b becomes insensitive to absorbtion if there is a second mode a. We consider two types of interactions: 1/ single photon absorbtion, 2/ two-photon and Raman type interactions.
1/ Let Hamiltonian of atoms and field has the form

$$H = H_P + H_F + V, \tag{8}$$

where H_P and $H_F = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b$ are Hamiltonians of free atoms and field, which describs by its operators of creation and annihilation of photons of the modes $a^\dagger, a$ and $b^\dagger, b$, with frequencies ω_a, ω_b. In dipole and rotating wave approximations operator of interaction between atoms and field takes the form

$$V = i\hbar[S_{10}(ga+fb) - S_{01}(ga+fb)^\dagger], \tag{9}$$

where $S_{xy} = \sum_m s_{xy}(m)$, $s_{xy}(m) = |x\rangle_m\langle y|$ is a single-atom operator, $x,y = 0,1$, $|0\rangle_m$, $|1\rangle_m$ are lower and upper level of atom, g,f are coupling constants which are assume to be real.
Introduce new variables

$$r = (ga+fb)G^{-1}, \tau = (fa-gb)G^{-1}, \tag{10}$$

where $G = \sqrt{g^2+f^2}$. Then the Hamiltonian of interaction (9) depends only on r: $V = i\hbar G[S_{10}r - S_{01}r^\dagger]$, and equation of motion for τ has the form

$$\frac{\partial}{\partial t}\tau = i\hbar^{-1}[H,\tau] = -i(f\omega_a a - g\omega_b b)/G. \tag{11}$$

It follows, that for the equal frequencies $\omega_a = \omega_b = \omega$ the linear combination of operators $\tau = (fa-gb)G^{-1}$ is integral of motion because of its evolution redices to the multiplication by a phase factor $\exp(-i\omega t)$.
Next points may be made about it. First, the found integral remains unchanged when atomic relaxation is taken into account by including, for example, in the Hamiltonian (8) an interaction between atoms and thermostat. Second, any function $\varphi(\tau,\tau^\dagger)$ is also integral of motion.
The presented model can describe the propagation of waves with identical frequencies resonant to the atomic transition. Fot this case, two waves can be distinguished by its polarization or direction of its propagation and simultaneously interact with the same atomic transition. These requirements are satisfied for, e.g., a broadband absorber that is insensitive to the light polarization. Further, we will consider absorption for definiteness. The unitary transformation U given by Eq. (10) selects the linear combination of the modes that does not interact with atoms. The transformation U can be realized by means of either a nonabsorbing beamsplitter or a parametric down convertor with the classical pumping

wave. Both physical systems are described by the effective Hamiltonian

$$\upsilon = i\hbar k(a^\dagger b - ab^\dagger), \quad (12)$$

where the modes a and b have the same frequencies and polarization for the case of the beamsplitter and they can have different frequencies and polarization in the case of the parametric down convertor.

For definiteness we will consider the beamsplitter, then one finds $U = \exp(-i\hbar^{-1}\upsilon t)$. Now introduce atomic relaxation by adding to the Hamiltonian H the operator of the interaction between atoms and thermostat V_E and Hamiltonian of the free thermostat H_E: $H \to H + H_E + V_E$. For simplicity, we consider only the resonance interaction between the modes a, b and atoms assuming that

$$\omega_a = \omega_b = \omega_0, \quad (13)$$

where ω_0 is the atomic transition frequency. Under this conditions we find

$$\begin{aligned} S(ga + fb) = \\ T\exp\{-i\hbar^{-1}\int_0^t dt'[V(ga+fb) + V_E(t')]\} \\ = U^\dagger T\exp\{-i\hbar^{-1}\int_0^t dt'[V(a) + V_E(t')]\}U \\ = U^\dagger S(a)U, \quad (14) \end{aligned}$$

where T is the time-order operator and we used the relation $U^\dagger aU = (ga + fb)G^{-1}$ together with $[U; V_E] = 0$. Equation (5) immediately follows from this relation.

Equation (14) enables to construct an optical scheme with two modes and atomic medium, where one of the modes is insensitive to absorption. To this end, two beamsplitters are placed in front of the absorber and behind it. The scheme operates as follows. Let the beamsplitters have the same transmittance and reflectance given by coefficients c and s, where $c = f/G, s = g/G$. Let a' and $, b'$ be the modes at the input of the absorbing medium and a'', b'' be these modes at the output of the absorber. Let the linear combination $\tau = (fa' - gb')/G$ be integral of motion, then $\tau = (fa'' - gb'')/G$. If modes a'' и b'' are mixed by the beamsplitter placed behind the absorber, then one finds the integral of motion $(ca'' - sb'')/G = b_{out}$ at one of the outputs of the beamsplitter, that is output of the scheme. It is convenient to consider the transformation of input modes a', b' in the inverse order. Let they be mixed at the beamsplitter placed in front of the absorber. In this case one finds integral of motion $(ca' - sb')/G = b_{in}$ at one of the outputs of the bemsplitter. But this output is one of the inputs of the scheme, then $b_{in} = \tau = b_{out}$. This means, that the mode b is reproduced at the end of the scheme being insensitive to absorbtion.

2/ Now consider a set of the unitary equivqlent schemes obtained by the the transformation (7), where interactions with classical waves are introduced. Let in Eq. (7) operator $N_B = 1$, and R_A is phase shift operator R_a of the mode a: $a \to a\exp(i\epsilon\Omega t)$, where $\epsilon = \pm 1$ and $\Omega > 0$. If $\epsilon = 0$, then one finds the previous case. All resources of the new schemes are obtained by the unitary transformation R_a of the initial resources, but the transformation leads to another physical processes. In particular, Hamiltonian of interaction (9) takes the form

$$V' = i\hbar[S_{10}(gae^{\epsilon i\Omega t} + fb) - S_{01}(ga^\dagger e^{-\epsilon i\Omega t} + fb^\dagger)]. \quad (15)$$

If $\epsilon = \pm 1$ the Hamiltonian describes the nondegenerate two photon absorbtion and Raman type interaction between the mode a, a strong classical wave at frequency Ω and atoms :

$$\begin{aligned} \omega_a + \Omega = \omega_b = \omega_0, \\ \omega_a - \Omega = \omega_b = \omega_0. \end{aligned} \quad (16)$$

From these relations new integral of motion $\tau' = (fa\exp(i\epsilon\Omega)t - gb)/G$ follows under the conditions that $\omega_a - \epsilon\Omega = \omega_b$, when Hamiltonian υ given by (12) is replaced to

$$\upsilon' = i\hbar k(a^\dagger b\exp(-i\epsilon\Omega t) - ab^\dagger\exp(i\epsilon\Omega t)). \quad (17)$$

This effective Hamiltonian describes the three-photon parametric process of up frequency conversion $\omega_a - \epsilon\Omega = \omega_b$ with the classical pumping wave at frequency Ω. As a result, to transmit the mode b through the absorbing medium in the obtained scheme it needs to mix it with the mode a in the parametric convertor, then to guide both modes to the absorber, and to separate the mode b using the second parametric convertor.

IV. THE QUANTUM CHANNEL FOR THE FOCK STATES OF LIGHT

As an example consider a problem of sending a Fock state through an absorber. According to Eqs. (5) and (9) the task can be accomplished by the scheme that consists of two beamsplitters. Assume the absorbing medium is described by the operator $S(ga + fb)$. We use Eq. (6), where the operator of evolution has the form $T_{AB} = US(ga + fb)U^\dagger$. We take the modes a and b in the coherent state $|\alpha\rangle$ and Fock state $|n\rangle$ respectively. Let the modes a and b be mixed on the first beamsplitter and come to the absorber at whose output the second beamsplitter is placed. According to Eq. (6), the Fock state can be reproduced at one of the outputs of the scheme and the coherent state, which is however weakened, can be found at the other output.

To describe the light, we use the normally ordered characteristic function C_N, which, in contrast to the Glauber-Sudarshan quasiprobability P, is a nonsingular function for the Fock states. The function C_N is defined as the mean value of the displacement operator being the Fourier transformation of the Glauber-Sudarshan quasiproba-

bility P:

$$C_N(\beta_1,\beta_2) = Sp\{\rho D_N(\beta_1) D_N(\beta_2)\}$$
$$= \int d^2\alpha_1 d^2\alpha_2 P(\alpha_1,\alpha_2) e^{\beta_1\alpha_1^*+\beta_2\alpha_2^*-\beta_1^*\alpha_1-\beta_2^*\alpha_2}, \quad (18)$$

where $D_N(\beta_k) = \exp(\beta_k c_k^\dagger)\exp(-\beta_k^* c_k)$, $k = 1,2, c_1 = a, c_2 = b$, and ρ is the density matrix of the modes a and b. If initially the modes a is in coherent state $|\alpha\rangle$ and b is in the Fock state $|n\rangle$ then the outgoing characteristic function of light has the form

$$C_N(\beta_1,\beta_2) = \exp\{(c\beta_1 - s\beta_2)\alpha^* - (c\beta_1 - s\beta_2)^*\alpha\} \sum_{k=0}^{n} C_k^n \frac{(-1)^k}{k!}|s\beta_1 + c\beta_2|^{2k}, \quad (19)$$

where c, s are transmittance and reflectance of the beam-splitters.

To describe propagating light trough the absorbing medium we will use the master equation for the field. This equation can be achieved with the help of adiabatic illumination procedure presented in [11] and the formalism of the quantum transfer theory [12]. For the simple case of the one dimensional problem in the Fokker-Planck approximation, the master equation has the form

$$(\partial/\partial t + v\partial/\partial z)C(z,t) = R\Big[((c - sf/g)\partial/\partial h + (s + cf/g)\partial/\partial e) (hc - hsf/g + se + ecf/g) + c.c.\Big]C(z,t), \quad (20)$$

where $R = g^2 N/\gamma$ is an absorbtion coefficient, N - is occupation of the lover level of atom, γ - is a transversal decay rate and

$$h = c\beta_1 - s\beta_2$$
$$e = s\beta_1 + c\beta_2. \quad (21)$$

To solve Eq. (20) it needs boundary conditions, which can be taken in the form (19), that corresponds to the light at the output of the first beamsplitter.

Let $sg = -cf$. Then all derivatives over e vanish and the solution has the form

$$C_N(z) = \exp[q(h\alpha^* - h^*\alpha)] \sum_{k=0}^{n} C_k^n \frac{(-1)^k}{k!}|e|^{2k}, \quad (22)$$

where $q = \exp(-Mz)$, $M = Rc^2(1 + (f/g)^2)$.

To achieve the state of light at the output of the absorber it needs changing the variables β_1, β_2 in (22) using (21). The unitary transformation of light by the second beamsplitter behind the absorber is described by replacing variables given by (21). It has the form $\beta_1' = c\beta_1 - s\beta_2, \beta_2' = s\beta_1 + c\beta_2$, where the beamsplitters are assume to be identical. As a result the outgoing characteristic function reads $C_N(\beta_1', \beta_2') = \exp[q(\beta_1'\alpha^* - \beta_1'^*\alpha)] \sum_{k=0}^{n} C_k^n (-1)^k |\beta_2'|^{2k}/k!$. It describes two independent modes in the coherent state $|q\alpha\rangle$ and in the Fock state $|n\rangle$ obtained by transformation

$$|\alpha\rangle \otimes |n\rangle \to |\alpha e^{-Mz}\rangle \otimes |n\rangle. \quad (23)$$

It tells, that the initial Fock state is reproduced at the output of the scheme and the amplitude of the coherent mode decreases.

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V. APPENDIX

We briefly discuss the properties of light obtained when the coherent state, which is usually associated with the wave nature of light, and the Fock state, which is associated with the corpuscular nature of light, are mixed by the beamsplitter. The characteristic function at the output of the beamsplitter, given by Eq. (19), describes the pure state

$$A_{n\alpha} = (1/\sqrt{n!})(sa + cb)^n A_{0\alpha}, \quad (24)$$
$$A_{0\alpha} = |c\alpha\rangle \otimes |-s\alpha\rangle,$$

where α and n are the complex amplitude of the coherent mode and the number of photons of the Fock mode. Thanks to the Fock state the nonclassical correlation of intensity of light aries and the sub-Poissonian statistics of photons can be found. Next two features are true: 1/ each of modes has sub-Poissonian statistics of photons and the Mandel parameter of the mode a, for example, reads $\xi_a = s^2n^2 2c^2(|\alpha|^2 - s^2)/(c^2|\alpha|^2 + s^2 n) \geq -1$; 2/ the joint photon coincidence count rate is lower than its value for a random flux $\langle a^\dagger a b^\dagger b\rangle - \langle a^\dagger a\rangle\langle b^\dagger b\rangle = -c^2s^2n(2|\alpha|^2+1)$; 3/ the variance of the photon number operator $u = a^\dagger a + \epsilon b^\dagger b$, where $\epsilon = \pm 1$, can be lower than the shot level $\langle a^\dagger a + b^\dagger b\rangle$ described the standart quantum limit. It means the suppression of the shot noise when the sum or difference of photocurrents of two detectors are measured.

Another class of correlations that are responsible for squeezing and entanglement is described by the quadrature operators of the modes, given by the operators of the canonical position and momentum. These correlations are measured in heterodyne schemes and they are phase sensitive. Because of the Fock state, which has not phase in the sense of the phase-space representation, the state $A_{n\alpha}$ is not squeezed.

Is the state $A_{n\alpha}$ entangled? It follows from (24) that the wave function is not factorized and in accordance with this property the state $A_{n\alpha}$ is entangled. For $n = 1$ the wave function $A_{1\alpha}$ reads $A_{1\alpha} = (1/\sqrt{2})(|t_0 t_1\rangle - |t_1, t_0\rangle)$, where $t_m = a^{\dagger m}|\alpha/\sqrt{2}\rangle$. It looks as EPR pair of discrete variables. In the same time because of the coherent state we can try to analyze entaglement with respect to continuous variables, using the cririon of non-separability [13].

According to this criterion for any separable state the inequality

$$C = \langle (\Delta Q)^2 \rangle + \langle (\Delta P)^2 \rangle \geq 2 \quad (25)$$

is valid, where the variances of the canonical operators of total position $Q = x_a + x_b$ and relative momentum $P = p_a - p_b$ are introduced and $a = x_a + ip_a, b = x_b + ip_b$. If this inequality is invalid, the question of the separability of the state is open. The reason is that this criterion is necessary and sufficient only for Gaussian fields which the Wigner function has the Gaussian form. In our case the Wigner function of $A_{n\alpha}$ is non-Gaussian, but it is really doesn't matter, because the inequality is valid:

$$C = 2(1 + n) \geq 2. \quad (26)$$

Thus the state $A_{n\alpha}$ is separable or unentangled. We remain open the problem of the relations between the factorizability of the wave function, entanglement and separability. However, the presented example shows that the state $A_{n\alpha}$ has opposite properties with respect to discrete and continuous variables. Nevertheless, the state can be used, for example, as a quantum channel for the standard protocol of teleportation of discrete variables. To prove this statement, it is sufficient to note that $A_{n\alpha}$ is unitary equivalent to a set of the Fock states which is a complete basis represented a Bell-like state measurement. Then new protocol is achieved by a simple local unitary transformation and a set of the recovering operators can be found. However, the separability of $A_{n\alpha}$ means that the state is at least free of the property of an EPR pair of continuous variables [14] for which $C = 0$.

[1] P. Zanardi, M. Rasetti, Phys. Rev. Lett. 79, 3306 (1997).

[2] A. M. Basharov, Pis'ma Zh. Eksp. Teor. Fiz. 75, 151 (2002) [JETP Lett. 75, 123 (2002)]; Zh. Eksp. Teor. Fiz. 121, 1249 (2002) [JETP 94, 1070 (2002)].

[3] . L.-M. Duan and G.-C. Guo, Phys. Rev. Lett. 79, 1953 (1997).

[4] D. A. Lidar, I. L. Chuang, K. B. Whaley, Phys. Rev. Lett. 81, 2594 (1998); D. A. Lidar, D. Bacon, J. Kempe, K. B. Whaley, Phys. Rev. A 61, 052307 (2000); J. Kempe, D. Bacon, D. A. Lidar, K. B. Whaley, Phys. Rev. A 63, 042307 (2001).

[5] M. Bourennane, M. Eibl, S. Gaertner, et al., Phys. Rev. Lett. 92, 107901 (2004).

[6] V. N. Gorbachev, A. I. Trubilko, Opt. Spektrosk. 89, 420 (2000) [Opt. Spectrosc. 89, 384 (2000)].

[7] V. N. Gorbachev and A. I. Trubilko, Opt. Spektrosk. 84, 970 (1998) [Opt. Spectrosc. 84, 879 (1998)].

[8] V. N. Gorbachev, A. I. Trubilko, Pis'ma Zh. Eksp. Teor. Fiz. 77, 563 (2003) [JETP Lett. 77, 469 (2003)].

[9] E. Knill, R. Laflemme, and G. J. Milburn, Nature 46, 409 (2001).

[10] A. I. Lvovsky, S. A. Babichev, Phys. Rev. A 66, 011801 (2002).

[11] V. N. Gorbachev, A. I. Zhiliba, Quantum Opt. 5, 193 (1993).

[12] V. N. Gorbachev and A. I. Zhiliba, J. Phys. A: Math. Gen. 33, 3771 (2000).

[13] L.-M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 84, 2722 (2000); R. Simon, Phys. Rev. Lett. 84, 2726 (2000).

[14] S. L. Braunstein and H. J. Kimble, Phys. Rev. Lett. 80, 869 (1998).